Multiphase Fluid Flow Simulator with Applications in Porous Media

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Phase-field Model

Cahn–Hilliard–Navier–Stokes System

- The unknown variables are **order parameter** c, **chemical potential** μ, **velocity** v, and **pressure** p.

$$\partial_t c - \nabla \cdot (M(c)\,\nabla\mu) + \nabla \cdot (cv) = 0 \text{ in } (0, T) \times \Omega,$$
$$\mu = \beta\,\Phi'(c) - \alpha\,\Delta c \text{ in } (0, T) \times \Omega,$$
$$\rho_0(\partial_t v + v \cdot \nabla v) - 2\nabla \cdot (\mu_s \varepsilon(v)) = -\nabla p + \mu\nabla c \text{ in } (0, T) \times \Omega,$$
$$\nabla \cdot v = 0 \text{ in } (0, T) \times \Omega.$$

- For flow through porous media scenarios, the model is supplemented with physically and mathematically **consistent boundary conditions** at the inlet and outlet.
- The relationship between **contact angle** and **surface tensions** is given by **Young's equation**.
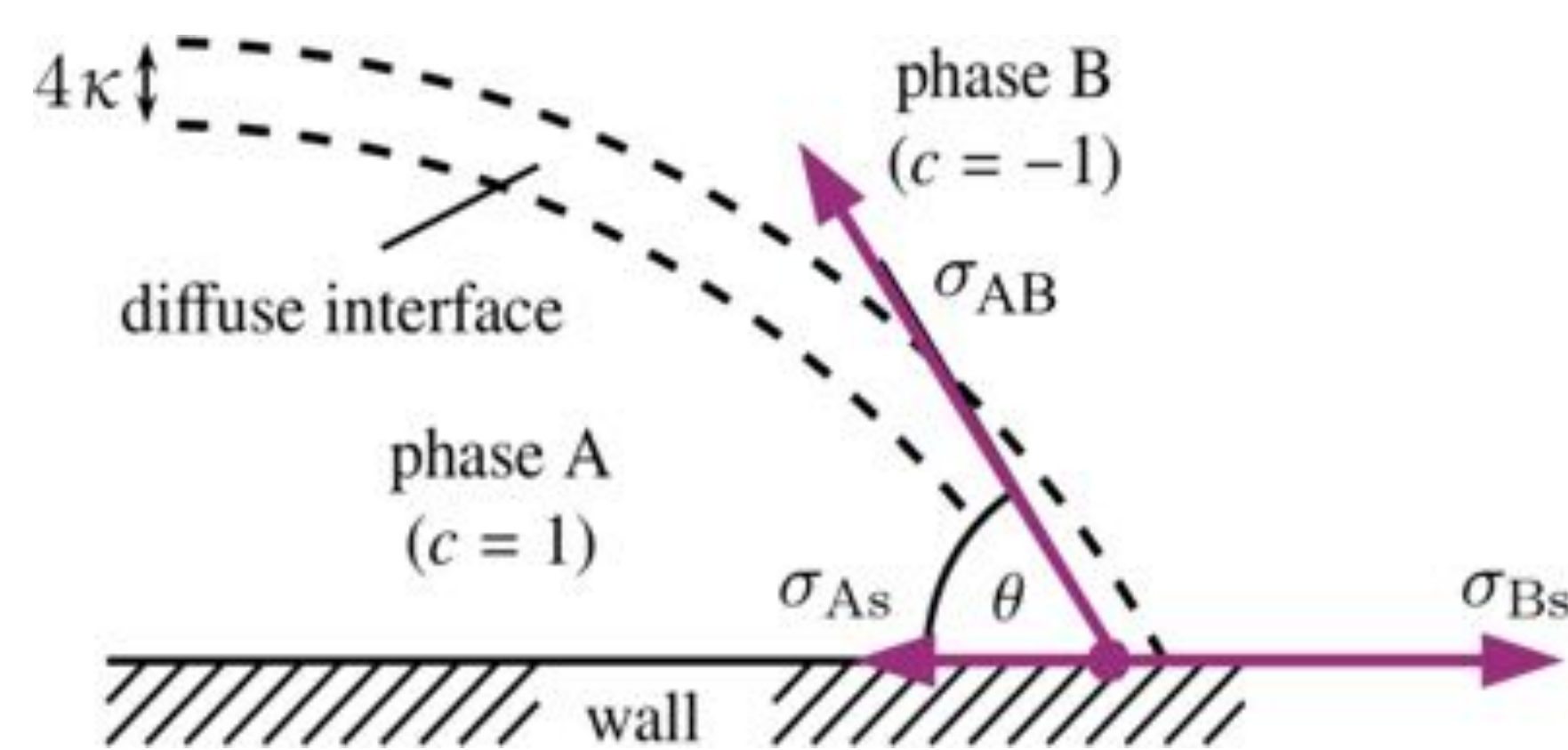
- Total energy F equals kinetic energy plus Helmholtz free energy plus surface energy:

$$F(c, v) = \int_\Omega \frac{\rho_0}{2}|v|^2 + \int_\Omega \left(\beta\,\Phi(c) + \frac{\alpha}{2}|\nabla c|^2\right) + \int_{\partial\Omega} \left((\sigma_{Bs} - \sigma_{As})g(c) + \sigma_{As}\right),$$

where $\Phi(c)$ denotes the **chemical energy density** and $g(c)$ denotes the **blending function**.

- Popular expressions of Φ include **Ginzburg–Landau potential** and **Flory–Huggins potential**.
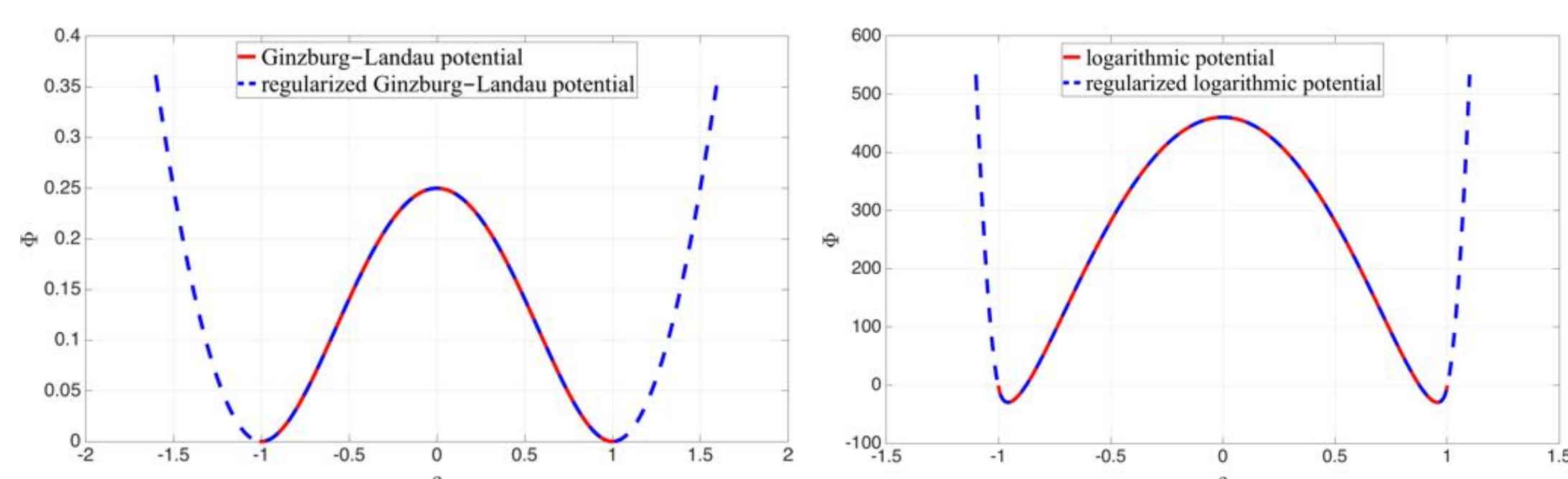
- The relationship between shear viscosity and order parameters is given by the **mixing rule**: linear rule, harmonic rule, or exponential rule.
- Provide four options for modeling **non-Newtonian** flows: power law model, Carreau model, Herschel–Bulkley model, and cross fluid model.
- A closed Cahn–Hilliard–Navier–Stokes system enjoys the properties of **mass conservation** and **energy dissipation**.

Merging Droplets
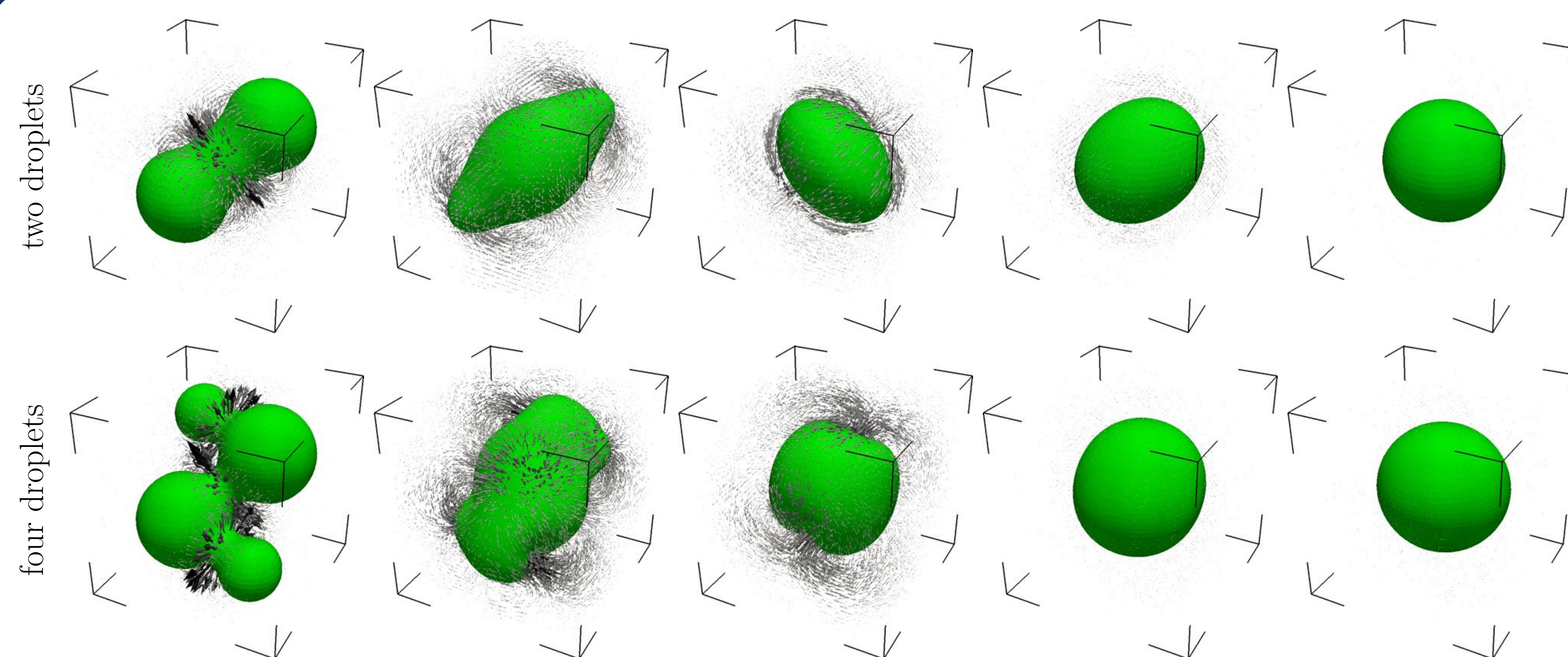
3D views of the evolution of the **order parameter** c (interface is shown in green and phase B is transparent) and the **velocity field** v. The droplets are merging together until a stationary state is reached.

Two-phase Flow in Berea Sandstone
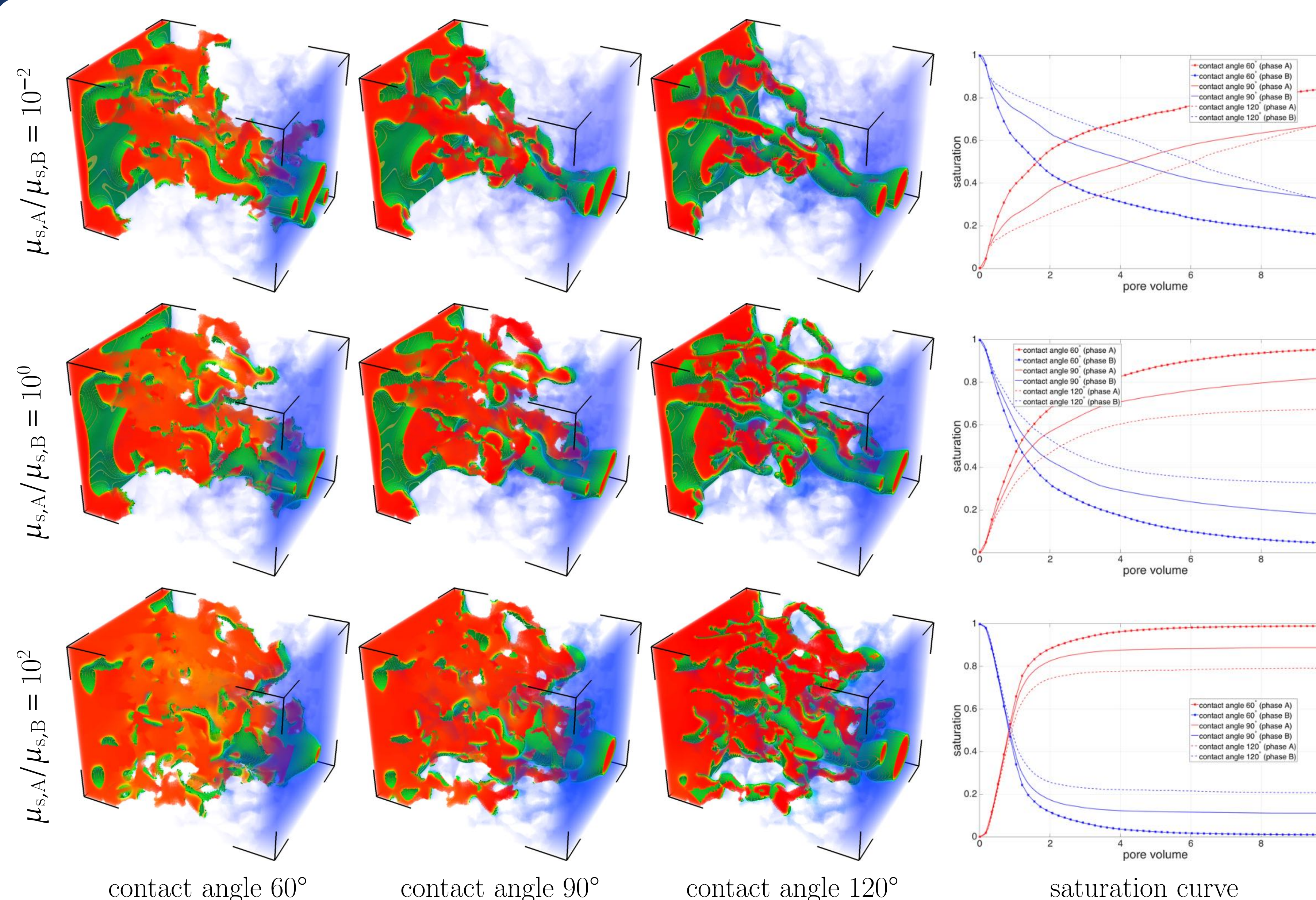
Micro-CT scan creates **porous images** at micrometer scale in which **voxel sets** represent the structure of porous media. The figure shows the order parameter field (red for phase A, green for the interface center, and blue / transparent for phase B).
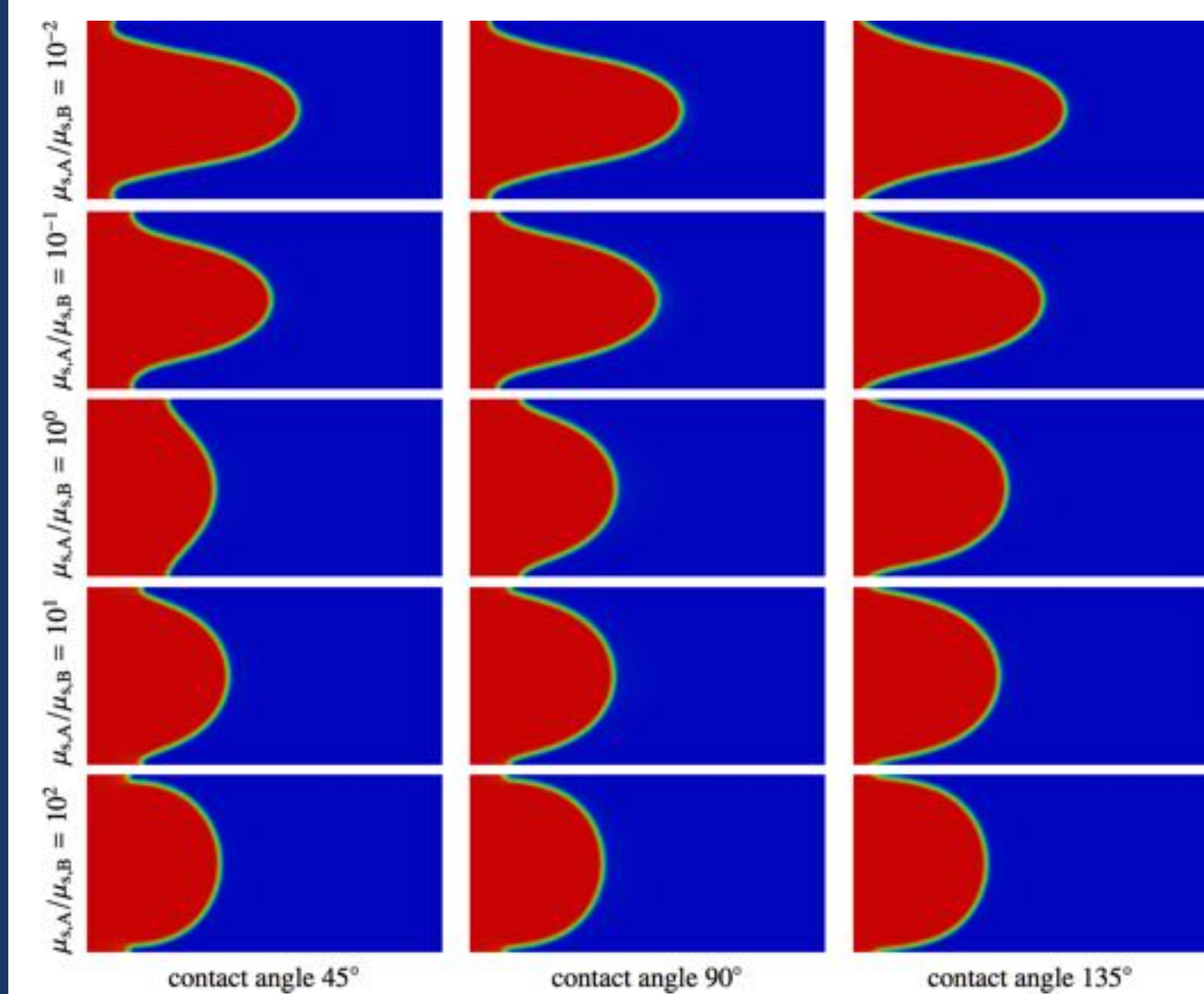
Numerical Method

- **Hierarchical bases** with orthogonal basis functions enable arbitrary order of approximation.
- **Interior penalty discontinuous Galerkin** methods for space discretization.
- **Implicit-explicit** scheme for time discretization.
- Rotational incremental **Pressure-correction projection** algorithm in conjunction with **div–div correction technique** ensures a pointwise solenoidal velocity field.
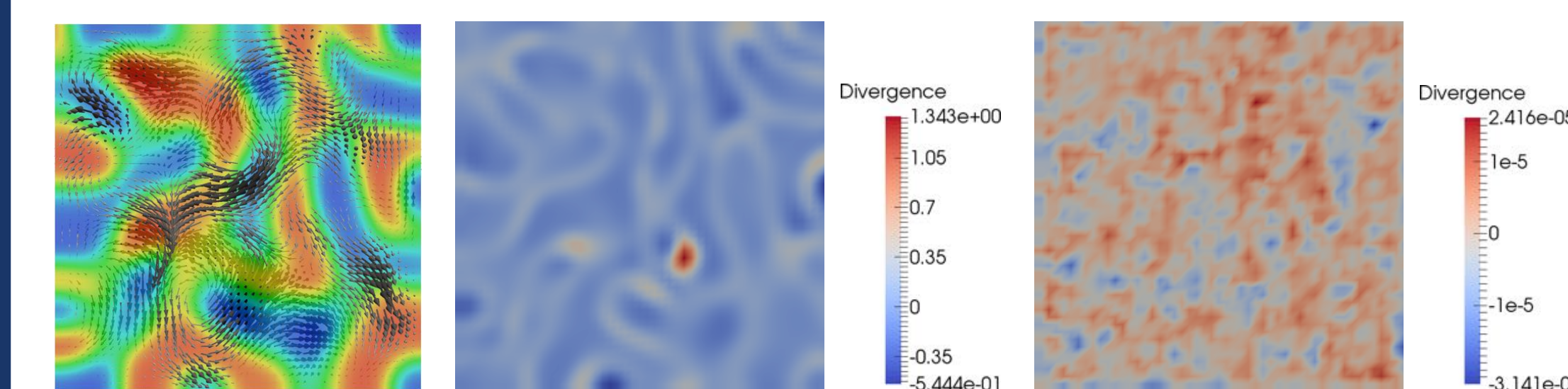
- Momentum balance equation: linearized by **Picard splitting**.
- Mass balance equation: linearized by **Newton's method**. Dissipates discrete free energy by utilizing a **convex–concave decomposition**. Scheme reduces to **cell-centered finite volumes** with the use of element-wise constants basis [1].

Cylindrical Pipe Simulations

Two-dimensional views of two-phase distribution in a cylinder.

[1] F. Frank, C. Liu, F. O. Alpak, S. Berg, and B. Rivière. "Direct numerical simulation of flow on pore-scale images using the phase-field method". In: *SPE Journal*, 23(5) (2018), pp. 1833–1850.